dent reports the rest take notes, just as they do when the instructor lectures. At the end of each report questions are asked and corrections are made. The notes taken by the rest of the students are corrected by the one who gives the report, and are bound up with the students' general note-book for the course. The one reporting binds up his outline, and a list of the books and papers consulted—a bibliography.

By this plan the student learns much about one animal not treated in the texts and he learns a little about a good many other species. But he does more—*he gets a training in using the powers of observation, in ordering the facts obtained and in expressing to others the knowledge gained.*

The two main suggestions are worth a trial by other teachers. The university should encourage the teaching of zoology by becoming a center for furnishing and distributing the material for the preparatory schools of a state at cost. Much of this could be secured very cheaply by a collecting expedition to Puget Sound. The student should be given the problem of furnishing the rest of the class with a report dealing with a special form of animal life somewhat closely related to a type studied. This working out of a "lecture?" by the student is the best of training for him.

W. J. BAUMGARTNER

SPECIAL ARTICLES

AN EXPRESSION FOR THE BENDING MOMENT AT ANY SUPPORT OF A CONTINUOUS GIRDER FOR ANY NUMBER OF EQUAL SPANS

TABLES giving the bending moments at the supports of a continuous uniformly loaded girder with equal spans are found in most of the books on strength of materials, but these tables usually stop at six or seven spans. The object of this paper is to give a general expression from which the bending moment at any support for any number of spans can be computed. First the expression and explanation of the method of computation are given and then follows the derivation of the formula.

Let M_1, M_2, $\cdots$ be the bending moments at the first, second ... support, respectively. Let n be the number of spans, w the load per unit length and l the length of span. If M_r represents the bending moment at the rth support then the formula gives

$$M_r = -\frac{\Delta_{r-2}D_{n-r+1} - D_{r-2}\Delta_{n-r}}{2\Delta_{n-1}}wl^2.$$

The Δs and Ds are numbers to be computed from the formulas.

$$\Delta_n = 4\Delta_{n-1} - \Delta_{n-2},$$
$$D_n = \Delta_{n-1} - D_{n-1}.$$

As shown below, $\Delta_0 = 1$, $\Delta_1 = 4$ and $D_0 = 0$ and any other Δ or D may be easily computed. For example,

$$\begin{aligned} \Delta_2 &= 4\Delta_1 - \Delta_0 = 15,\\ \Delta_3 &= 4\Delta_2 - \Delta_1 = 56,\\ &\cdots\cdots\\ D_1 &= \Delta_0 - D_0 = 1,\\ D_2 &= \Delta_1 - D_1 = 3. \end{aligned}$$

Thus, if, for example, we wish the bending moment at the fourth support for seven spans, we have $r = 4$, $n = 7$ and

$$M_4 = -\frac{\Delta_2 D_4 - D_2\Delta_3}{2\Delta_6}wl^2.$$

From the above formulas $\Delta_2 = 15$, $D_4 = 44$, $D_2 = 3$, $\Delta_3 = 56$, $\Delta_6 = 2911$. Hence

$$[M_4]_{7\ \text{spans}} = -6/71\,wl^2,$$

a result which is verified by the tables.

The derivation of the above formula is nothing but the general solution of the equations of three moments by determinants. For n spans we have, from the theorem of three moments,

$$\begin{aligned} M_1 + 4M_2 + M_3 &= -wl^2/2,\\ M_2 + 4M_3 + M_4 &= -wl^2/2,\\ &\cdots\cdots\\ M_{n-1} + 4M_n + M_{n+1} &= -wl^2/2. \end{aligned}$$

Since $M_1 = M_{n+1} = 0$ we have left $n-1$ equations with $n-1$ unknowns. If we write 1 in place of $-wl^2/2$ and multiply the final result by $-wl^2/2$ the solution will be less complicated. Writing the Ms with the same subscripts under one another we have

$$\begin{aligned} 4M_2 + M_3 \qquad\qquad &= 1,\\ M_2 + 4M_3 + M_4 \qquad &= 1,\\ M_3 + 4M_4 + M_5 &= 1,\\ \cdots\cdots\cdots& \end{aligned}$$

The determinant of the system of equations will be the determinant,

$$\begin{vmatrix} 4 & 1 & 0 & 0 & 0 & \cdots \\ 1 & 4 & 1 & 0 & 0 & \cdots \\ 0 & 1 & 4 & 1 & 0 & \cdots \\ 0 & 0 & 1 & 4 & 1 & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdots \end{vmatrix}$$

of order $n-1$. We will represent it by Δ_{n-1}. The solution of the system of equations for any unknown, say M_r, will be a fraction with Δ_{n-1} for the denominator. The numerator of the fraction will be a determinant of order $n-1$ with the same elements as Δ_{n-1} except that each element in the $r-1$th column is 1. By expanding Δ_{n-1} it is easy to see that the general formula

$$\Delta_n = 4\Delta_{n-1} - \Delta_{n-2}$$

holds. Since $\Delta_1 = 4$ and Δ_0 may be defined as 1, any Δ may be computed.

For computing the determinant in the numerator we let D_n represent a determinant of the nth order which has the same elements as Δ_n except that each element of the first column is 1. Expanding D_n, it is found that

$$D_n = \Delta_{n-1} - D_{n-1}.$$

D_0 is to be defined as 0. Now expanding the numerator of the fraction representing M_r in terms of minors of the upper $r-2$ rows, we find

$$M_r = \frac{\Delta_{r-2}D_{n-r+1} - D_{r-2}\Delta_{n-r}}{\Delta_{n-1}},$$

and multiplying this result by $-wl^2/2$ we have the general expression given at the beginning of this article. In computing a table from this formula it is of course not necessary to compute all the Ms, for the bending moments at supports equidistant from the ends are equal, that is,

$$M_r = M_{n-r+2}.$$

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SOCIETIES AND ACADEMIES

THE BOTANICAL SOCIETY OF WASHINGTON

THE sixtieth regular meeting of the society was held at the Ebbitt House, February 19, 1910, at eight o'clock P.M.; President Wm. A. Taylor presided. The following papers were read:

Sprout Leaves of Western Willows: C. R. BALL, U. S. Bureau of Plant Industry.

A knowledge of the range of variation in the leaves of willows is important because a large proportion of the herbarium material must be determined from foliage specimens only. This is due to the precocious flowering of many species and the quick disappearance of the staminate aments from all, thus leaving fully half the plants in this diœcious genus with only the leaves as determining features. The pistillate aments also are gone from plants of the diœcious species before most collectors reach the field. The leaves of the so-called water sprouts are interesting because of their wide departure from the normal, especially in size and to some extent in form also.

A series of collections shows that the proportion of breadth to length found in the normal leaves is maintained in sprout leaves from the same individual in several species of the sections Pentandræ, Longifoliæ and Cordatæ from the western United States. A variation of form was found in a specimen of *S. scouleriana* (section Capreæ) from Arizona, in which the normal leaves are obovate, but those of this sprout were broadly ovate. The paper was illustrated by numerous specimens.

Bull-horn Acacias in Botanical Literature, with a Description of two new Species: W. E. SAFFORD, U. S. Bureau of Plant Industry.

There has been much confusion as to the identity of certain acacias of Mexico and Central America having large inflated horn-like stipular thorns, which are usually inhabited by ants. Linnæus placed all which had been described previously to the publication of his "Species Plantarum," under a single species *Mimosa cornigera.* Schlechtendal and Chamisso recognized the fact that the supposed synonyms cited by Linnæus included more than one species. These authors described two species found in the collections of Schiede from the state of Vera Cruz, Mexico, which they named *A. spadicigera* and *A. sphærocephala.* They leave it in doubt whether either of these species is the *Arbor cornigera,* figured and described by Hernandez (ed. Rom., p. 86, 1656), which in all probability is identical with the first plant cited by Linnæus, under his description of *Mimosa cornigera.*

In the National Herbarium are specimens of a bull-horn acacia from the type region of Hernandez's plant, collected by Dr. Edward Palmer. There are also at least two others quite distinct from any species hitherto described, one of them from Guatemala, with the inflorescence in spherical heads and with very long slender dehiscent pods; the other from the state of Chiapas, southern Mexico, with spadix-like inflorescence and